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Note on shadowing and diffraction in deep-inelastic lepton scattering*

G. Piller[1], L. Ferreira[2], W. Weise[1]

[1] Physik Department, Technische Universität München, D-85747 Garching, Germany
[2] Departamento de Fisica and CFIF-Edificio Ciencia, Instituto Superior Tecnico, P-1096 Lisboa, Portugal

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Abstract. We discuss the close relation between shadowing in deep-inelastic lepton-nucleus scattering and diffractive photo- and leptoproduction of hadrons from free nucleons. We show that the magnitude of nuclear shadowing at small Bjorken-x, as measured by the E665 and NMC collaborations, is directly related to HERA data on the amount of diffraction in the scattering from free nucleons.

In recent years diffractive photo- and leptoproduction processes have received much attention. This interest has been initiated mainly by the large amount of new data from the HERA electron-proton collider (for reviews see [1,2]). While diffraction from free nucleons is an interesting subject all by itself, it also plays a major role in the shadowing phenomena observed in deep-inelastic lepton scattering from nuclei. Here plenty of data have become available in the last decade from high precision experiments at CERN (NMC) [3,4] and FNAL (E665) [5,6].

In this note we exploit the close relation between shadowing and diffraction [7]. The connection of these two phenomena is well known and used frequently in the literature. Nevertheless, it is instructive to see that the magnitude of nuclear shadowing in deep-inelastic lepton scattering is determined in a simple and basic way by the relative amount of diffraction observed in the high-energy scattering from free nucleons.

In deep-inelastic lepton scattering from nuclei shadowing occurs at small values of the Bjorken scaling variable $x = Q^2/2p\cdot q$, where p^μ and q^μ are the nucleon and photon four-momenta, respectively, and $Q^2 = -q^2$. In the following we choose the photon momentum in the $\hat{z}$-direction, i.e. $\boldsymbol{q} = (\boldsymbol{0}_\perp, q_z)$. At $x < 0.1$ hadronic components of the photon field can interact coherently with several nucleons in the nuclear target. Destructive interference of multiple scattering amplitudes leads to a depletion (shadowing) of nuclear structure functions F_2^{A} as compared to A times the one for free nucleons, F_2^{N}. The same observation holds, of course, for the virtual photon-nucleus cross section $\sigma_{\gamma^*\mathrm{A}} = (4\pi^2\alpha/Q^2)\, F_2^A$.

This cross section can be separated into a piece which accounts for the incoherent scattering from individual nucleons, and a correction due to the coherent interaction with several nucleons, $\sigma_{\gamma^*\mathrm{A}} = \mathrm{A}\,\sigma_{\gamma^*\mathrm{N}} + \delta\sigma_{\gamma^*\mathrm{A}}$. The multiple scattering correction $\delta\sigma_{\gamma^*\mathrm{A}}$ is dominated by contributions which involve two nucleons. In this note we restrict ourselves to this so-called double scattering correction[1]. It is controlled by the diffractive production of hadrons from single nucleons and their subsequent rescattering. This process, illustrated in Fig.1, is described by the well known relation [7]:

$$\begin{aligned}\delta\sigma_{\gamma^*\mathrm{A}} \approx -8\pi \int_{4m_\pi^2}^{W^2} dM_{\mathrm{X}}^2 \int d^2b \int_{-\infty}^{+\infty} dz_1 \\ \times \int_{z_1}^{+\infty} dz_2\, \rho_{\mathrm{A}}(\boldsymbol{b}, z_1)\rho_A(\boldsymbol{b}, z_2) \\ \times \cos\left[(z_2 - z_1)/\lambda\right] \left.\frac{d^2\sigma_{\gamma^*\mathrm{N}}^{diff}}{dM_{\mathrm{X}}^2 dt}\right|_{t\approx 0}.\end{aligned} \quad (1)$$

A state X with invariant mass M_{X} is produced diffractively in the process $\gamma^*\mathrm{N} \to \mathrm{XN}$, with the nucleon located at $(\boldsymbol{b}, z_1)$. The upper limit of M_{X} is determined by the available $\gamma^*\mathrm{N}$ center-of-mass energy W. The hadronic excitation propagates at fixed impact parameter $\boldsymbol{b}$ and then interacts with a second nucleon at z_2. The underlying basic mechanism, i.e. diffraction from a single nucleon, is described by the cross section $d^2\sigma_{\gamma^*\mathrm{N}}^{diff}/dM_{\mathrm{X}}^2dt$ taken in the forward direction, $t = (p-p')^2 \approx 0$, where p' is the four-momentum of the scattered nucleon. The product of nu-

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[1] Corrections due to triple scattering amount typically to less than 30% of the double scattering term [8].

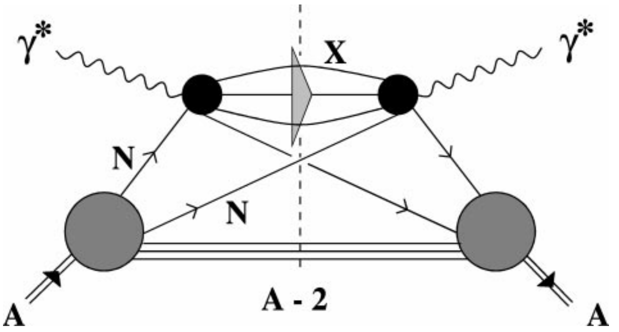

Fig. 1. Double scattering contribution to deep-inelastic charged lepton scattering from nuclei

clear densities, $\rho_\mathrm{A}(\boldsymbol{b}, z_1)\, \rho_\mathrm{A}(\boldsymbol{b}, z_2)$, accounts for the probability to find two nucleons in the target at the same impact parameter. We use the normalization $\int d^3r\, \rho_\mathrm{A}(\boldsymbol{r}) = A$. Finally, the $\cos[(z_2 - z_1)/\lambda]$ factor in (1) implies that only diffractively excited hadrons with a longitudinal propagation length larger than the average nucleon-nucleon distance in the target,

$$\lambda = \frac{2\nu}{Q^2 + M_\mathrm{X}^2} > d \approx 2\,\mathrm{fm}, \tag{2}$$

can contribute significantly to double scattering.

Equation (1) has been applied in several investigations of nuclear shadowing, using different models for the diffractive photoproduction cross section (see e.g. [8–15]). A simple estimate of nuclear shadowing at small Bjorken-x can, however, already be obtained by just looking at the relative amount of diffractive production in deep-inelastic scattering from free nucleons which is known from recent experiments at HERA.

For $x \ll 0.1$, the coherence lengths λ of the hadronic states which dominate diffractive production in (1), exceed the diameter of the target nucleus. In the limit $\lambda \to \infty$ we find:

$$\delta\sigma_{\gamma^*\mathrm{A}} \simeq -8\pi\, B\, \sigma_{\gamma^*\mathrm{N}}^{diff} \times \int d^2b \int_{-\infty}^{+\infty} dz_1 \int_{z_1}^{+\infty} dz_2\, \rho_\mathrm{A}(\boldsymbol{b}, z_1)\, \rho_\mathrm{A}(\boldsymbol{b}, z_2). \tag{3}$$

The slope parameter B describes the t-dependence of the diffractive production cross section

$$\frac{d^2\sigma_{\gamma^*\mathrm{N}}^{diff}}{dM_\mathrm{X}^2 dt} = e^{-B|t|} \left. \frac{d^2\sigma_{\gamma^*\mathrm{N}}^{diff}}{dM_\mathrm{X}^2 dt} \right|_{t=0}. \tag{4}$$

Recent HERA data on the integrated diffractive leptoproduction cross section are well described using $B \simeq 7$ GeV^{-2} [16]. In the diffractive production of low mass vector mesons (ρ, ω and ϕ) from nucleons, a range of values $B \simeq (4 \ldots 10)$ GeV^{-2} has been found, depending on Q^2 and on the incident photon energy (for reviews see e.g. [17]).

For the nuclear densities in (3) we use Gaussian,

$$\rho_\mathrm{A}(\boldsymbol{r}) = A \left(\frac{3}{2\,\pi\,\langle r^2\rangle_\mathrm{A}}\right)^{3/2} \exp\left(-\frac{3\,\boldsymbol{r}^2}{2\,\langle r^2\rangle_\mathrm{A}}\right), \tag{5}$$

and square-well parametrizations,

$$\rho_\mathrm{A}(\boldsymbol{r}) = \begin{cases} A \frac{3}{4\pi} \left(\frac{3}{5\,\langle r^2\rangle_\mathrm{A}}\right)^{3/2} & \text{for} \quad r < \sqrt{\frac{5}{3}}\, \langle r^2\rangle_\mathrm{A}^{1/2}, \\ 0 & \text{otherwise}, \end{cases} \tag{6}$$

with the mean square radius $\langle r^2\rangle_\mathrm{A} = \int d^3r\, r^2\, \rho_\mathrm{A}(r)/A$. For both cases the shadowing ratio $R_\mathrm{A} = \sigma_{\gamma^*\mathrm{A}}/A\sigma_{\gamma^*\mathrm{N}}$ is easily worked out:

$$R_\mathrm{A} \simeq 1 - \mathcal{C}\, A \left(\frac{B}{\langle r^2\rangle_\mathrm{A}}\right) \frac{\sigma_{\gamma^*\mathrm{N}}^{diff}}{\sigma_{\gamma^*\mathrm{N}}}. \tag{7}$$

For Gaussian nuclear densities one finds $\mathcal{C} = 3$, while $\mathcal{C} = 2.7$ in the square-well case.

To estimate the magnitude of nuclear shadowing as observed by NMC and E665 one needs to know the diffraction part of the free nucleon cross section for comparable kinematic conditions. Recent data on diffractive production are available from HERA [18–20] for incident real photons. It has been found that diffractive processes, which leave the target proton intact, amount to approximately 20% of the total photon-nucleon cross section at center-of-mass energies $W \sim 200$ GeV. About half of the diffractive events are due to vector meson production. In leptoproduction at large $Q^2 \sim 10$ GeV^2, on the other hand, diffractive production reduces to about 10% of the total deep-inelastic scattering cross section [21].

In fixed target lepton scattering experiments the average squared momentum transfer $\overline{Q}^2$ decreases for decreasing x. For the NMC and E665 experiments one finds, for example, $\overline{Q}^2 \lesssim 0.4$ GeV^2 at $x \lesssim 10^{-3}$ [4,6]. In this high-energy region it is legitimate to assume Regge behavior for the energy dependence of diffraction (see e.g. [20]). At $x \sim 10^{-4}$ the center-of-mass energies in the NMC and E665 measurements are $W \sim 15$ GeV [4] and $W \sim 25$ GeV [6], respectively. For these energies the relative amount of diffraction reaches about 60% of the value found at HERA. Note that this agrees within 25% with an analysis of earlier data on diffraction from FNAL [22].

In our estimate (7) we use $B \approx 8$ GeV^{-2} for the slope parameter. This is a reasonable average of the corresponding values extracted from diffractive photoproduction of low mass vector mesons [17,22] and high mass states [16, 22]. Using furthermore $\sigma_{\gamma^*\mathrm{N}}^{diff}/\sigma_{\gamma^*\mathrm{N}} \simeq 0.1$ the magnitude of R_A comes out in very good agreement with experimental values as shown in Table 1. This estimate confirms in a simple and basic way that shadowing in nuclear deep-inelastic scattering is governed by the coherent interaction of diffractively produced states with several nucleons in the target nucleus.

Note that shadowing and diffraction are also linked in deep-inelastic charged current interactions. Nuclear shadowing and diffraction have been observed here too, although with large experimental errors (for a review see e.g. [23]). A detailed theoretical investigation of shadowing effects in neutrino-nucleus scattering has been carried out recently in [24]. It is found that shadowing in charged lepton and neutrino scattering is of similar magnitude.

Table 1. The shadowing ratio R_{A} estimated according to (7) in comparison with experimental data for various nuclei. Here square-well nuclear densities have been used. The data are taken from [5,6,4] at the smallest kinematically accessible values of the Bjorken variable ($x \simeq 10^{-4}$)

	^{6}Li	^{12}C	^{40}Ca	^{131}Xe
R_{A}	0.93	0.84	0.73	0.65
$R_{\mathrm{A}}^{exp.}$	0.94 ± 0.07	0.87 ± 0.10	0.77 ± 0.07	0.67 ± 0.09

Then the relative amount of diffraction also is expected to be roughly the same.

To summarize: based on the simple geometric relation (7) we have illustrated how the magnitude of nuclear shadowing at $x \simeq 10^{-4}$, as measured by the NMC and E665 collaborations, is directly and very easily connected to the relative amount of diffraction from free nucleons which has been determined recently at HERA. This close connection emphasizes the role of diffraction in nuclear deep-inelastic scattering at small Bjorken-x, and (7) proves useful for practical estimates of the leading coherent double scattering effect.

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